Towards a multi-qubit concurrence witness

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Abstract:

Many-body entanglement is a generic feature of many-body dynamics. As a resource, it is plays a central role in, for instance, quantum computing, communication and metrology [1,2]. Quantification of this resource is thus important for such applications. The “concurrence” is a measure of multi-partite entanglement. It is related to the entanglement of formation, and is defined via anti-unitary transformations of the quantum state [3]. Anti-unitaries are challenging to implement experimentally. We aim to develop protocols which can witness or measure concurrence in state-of-the-art quantum simulators.

[1] Horodecki et al. , Rev. Mod. Phys. **81** (2009)

[2] Amico et al. , Rev. Mod. Phys. **80** (2008)

[3] A. Uhlmann, PRA, **62**, 032307

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Concurrence of N qubits

$$\begin{aligned}\mho(\alpha|\psi\rangle + \beta|\phi\rangle) &= \alpha^*\mho|\psi\rangle + \beta^*\mho|\phi\rangle,\\ \mho^\dagger &= \mho^{-1}\end{aligned}$$

$\tilde{\rho} = \mho\rho\mho^\dagger$, with $\mho = (-i\sigma_y)^{\otimes N}\mathcal{K}$ an anti-unitary.

$$C_N(\rho) = \begin{cases} \sqrt{\rho\tilde{\rho}} = \left|\langle\psi|\tilde{\psi}\rangle\right| = \mathcal{F}(|\psi\rangle, |\tilde{\psi}\rangle) & \text{for } \mathcal{P}(\rho) = 1,\\ \max\left\{0, \lambda_1 - \sum_{j\geq 2}\lambda_j\right\} = \max\left\{0, 2\lambda_1 - \mathrm{tr}\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}\right\} & \text{for } \mathcal{P}(\rho) \neq 1,\\ 0 & \text{for } N \text{ odd.}\end{cases}$$

Time-reversal

Can it be measured?

$$\rho(\tilde{t}) = \mho e^{-iHt}\rho_0 e^{iHt}\mho^\dagger = e^{+i\mho H\mho^\dagger t}\mho\rho_0\mho^\dagger e^{-i\mho H\mho^\dagger t}$$

- Time-reversed dynamics are possible [4]
- Time-reversal of state?
  - Real pure states - use an ancilla [5]
  - Thermal states of TRS Hamiltonians [5]
  - Full state tomography (FST): $\sim 4^N$ parameters
  - Embedded Quantum Simulators: Can implement $\mathcal{K}$ [6]

Relation to other measures

- Entanglement of formation (2 qubits)
- Quantum Fisher information (n-qubit GHZ-like state)
- Multiple Quantum Coherence (2 qubits)

Our goal:

Useful witness via measurable lower bound.

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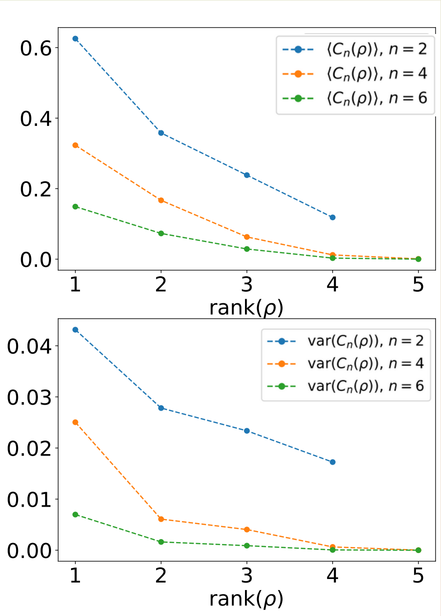
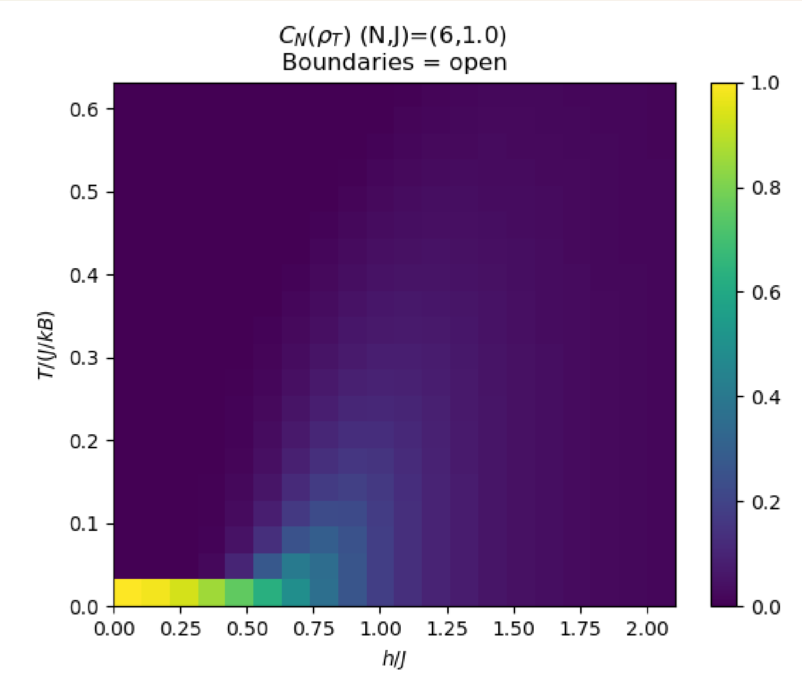
Concurrence vs purity

Random mixed states

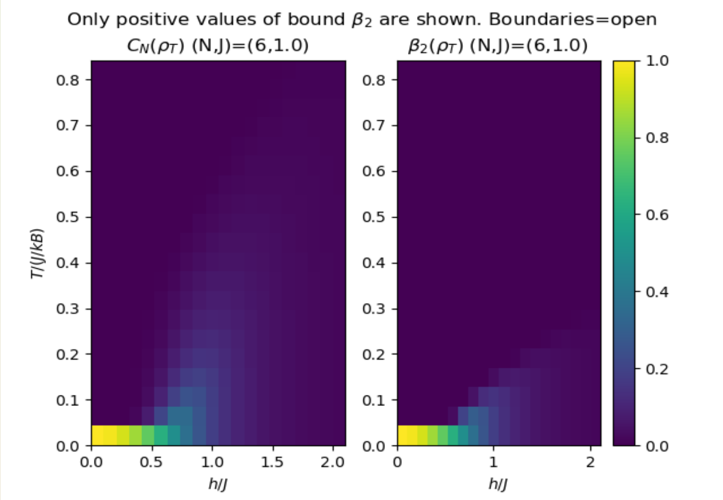


Transverse-field Ising model



Candidate for a measurable lower bound

Using sub- and super-fidelity [7] we find $\beta(\lambda_1, \mathcal{P}(\rho), \mathrm{tr}\rho\tilde{\rho}) = 2\lambda_1 - \sqrt{\mathrm{tr}\rho\tilde{\rho} - (1 - \mathcal{P}(\rho))} \leq C_N(\rho)$



Recall $\lambda_1 = \max\mathrm{spec}(\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}})$

Outlook

- How to measure $\lambda_1, \mathrm{tr}\rho\tilde{\rho},$ and $\mathcal{P}(\rho)$?
- Can we use “random measurement protocols”?
  - $\mathcal{P}(\rho)$ and λ_1 [9]
  - $\mathrm{tr}\rho r\tilde{h}o = \mathrm{tr}\rho\mathfrak{S}\rho\mathfrak{S}^{-1}$ [10]

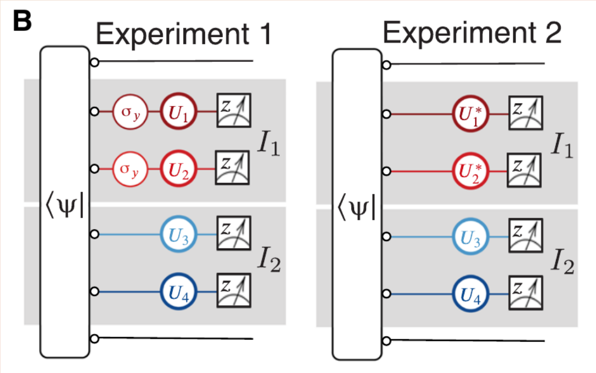


Figure from [9]

References

[4] Gärttner et al., Nat. Phys. **13** (2017); [5] Brennen et al., PRA **70** (2004)
[6] Zhang et al., Nat. Commun. **6** (2015);
[7] Miszczak et al. , Quant. Info. and Comp. **9** (2009);
[8] Pichler et al., PRX **6** (2016); [9] van Enk et al., PRL **108** (2012)
[10] Elben et al., Sci. Adv. 2020; **6**

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