Linear and non-linear edge dynamics of small bosonic FQH droplets

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Introduction

A gas of rapidly rotating bosons can enter strongly correlated phases which are the bosonic versions of fractional quantum Hall states [1]. We study the linear and nonlinear dynamics of the chiral edge excitations of small bosonic droplets in a fractional quantum Hall state away from the chiral Luttinger liquid limit [2]. This regime is of interest because non-dissipative density shock waves are expected to take place due to velocity gradients in a propagating deformation, meaning that the edge density will at some point try to "overturn". This tendency is regularized by quantum effects which produce highly non-linear oscillatory features in the density, which have been theorised, by inspecting 1D models, to further evolve into a series of localized pulses carrying "fractional-charge" [3, 4].

Rapid rotation limit

We consider a 2D bosonic gas with contact interactions in a axisymmetric trap; in the frame rotating about the $\hat{z}$-axis with rotation frequency Ω the Hamiltonian reads [1]

$$\begin{cases} H_\Omega = \sum_i h_i + g \sum_{j<i} \delta^{(2)}(\mathbf{x}_i - \mathbf{x}_j) \\ h = \frac{1}{2m}(\mathbf{p} - m\mathbf{\Omega} \times \mathbf{r})^2 + \frac{1}{2}m(\omega^2 - \Omega^2)r^2. \end{cases} \tag{1}$$

Here the Coriolis force has the same form as the Lorentz force on a charged particle in a uniform magnetic field. Close to the centrifugal limit $\Omega \approx \omega$, the *exact* groundstate of a N boson system is the bosonic Laughlin wavefunction

$$\psi(\{z_i\}) = \prod_{i<j}(z_i - z_j)^2 \exp\left(-\frac{|z_i|^2}{4l^2}\right), \tag{2}$$

with angular momentum $L = N(N-1)$. The properties are analogous to the fermionic counterparts: uniform bulk density, gapped collective bulk excitations and gapless edge modes with $L > N(N-1)$ which propagate chirally. In a harmonic trap these modes are heavily degenerate and their spectrum is linear: edge excitation propagate at speed $v_\theta = \frac{1}{2}\left(\frac{\omega^2}{\Omega^2} - 1\right)$. In this limit the χLL is exact.

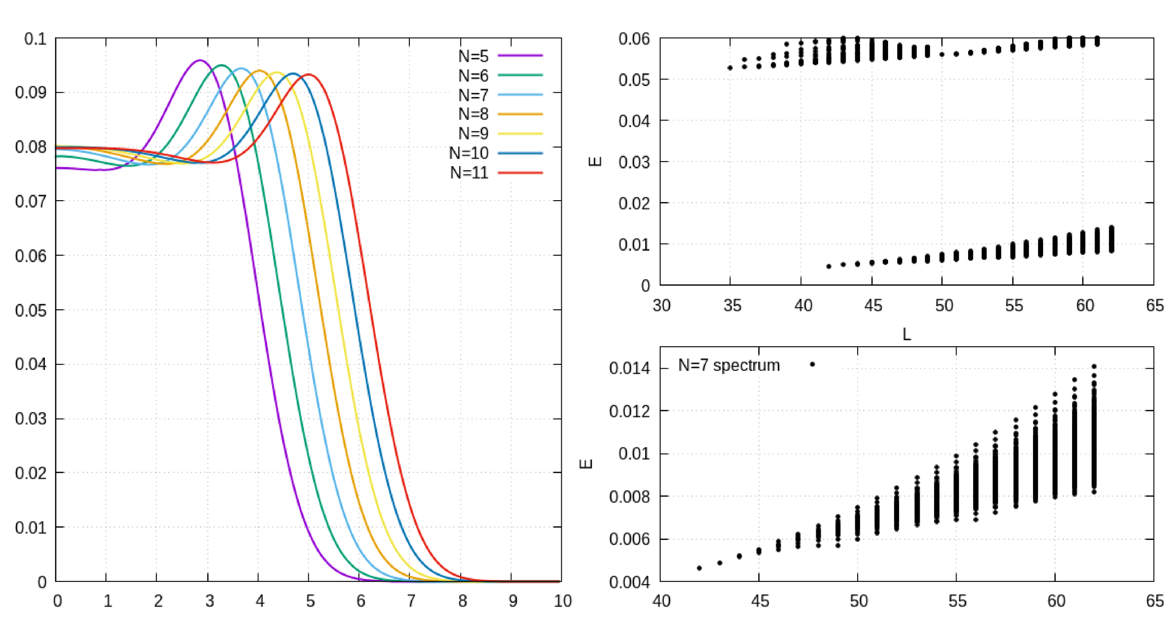

Fig. 1: Laughlin ground state - densities for different N and many-body spectrum at $N = 7$.

Introduction of weak axisymmetric anharmonicity splits the degeneracy and introduces relevant non-linear features in the many-body dynamics.

Linear response

The "charge" that builds up on the edges when perturbing the system in a time-dependent fashion is the results of the quantized Hall current in the bulk $J_r \propto \nu \partial_\theta U$, proportional to the filling fraction. One can therefore expect a factor $\approx \frac{\nu'}{\nu}$ between two different quantum Hall phases.

The results for bosonic droplets in the $\nu = \frac{1}{2}$ state are compared with those for non-interacting fermions at filling $\nu = 1$. Remarkably the amplitude of the edge modulation of the FQH state converges to half of that of a large $\nu = 1$ system already when the number of particles involved is ≈ 10.

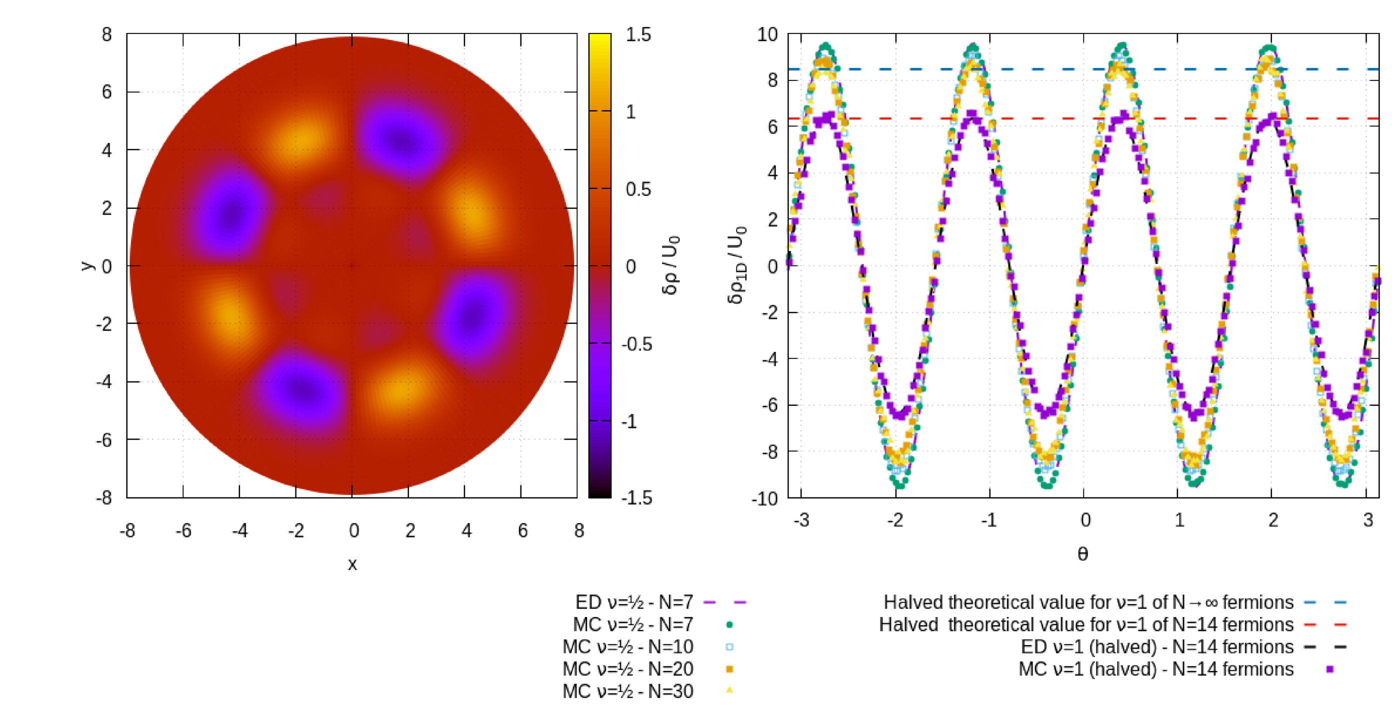

Fig. 2: Response to a weak linear excitation - densty difference with respect to the Laughlin state and integrated density.

Interestingly, the presence of the density overshoot [5] in the $m = 2$ Laughlin state shows up in the (2D) density dynamics too. This is not the case at unit filling factor, where the density bump at the edge comes solely from the confining potential and is not a many-body feature of the system.

Non-linear dynamics

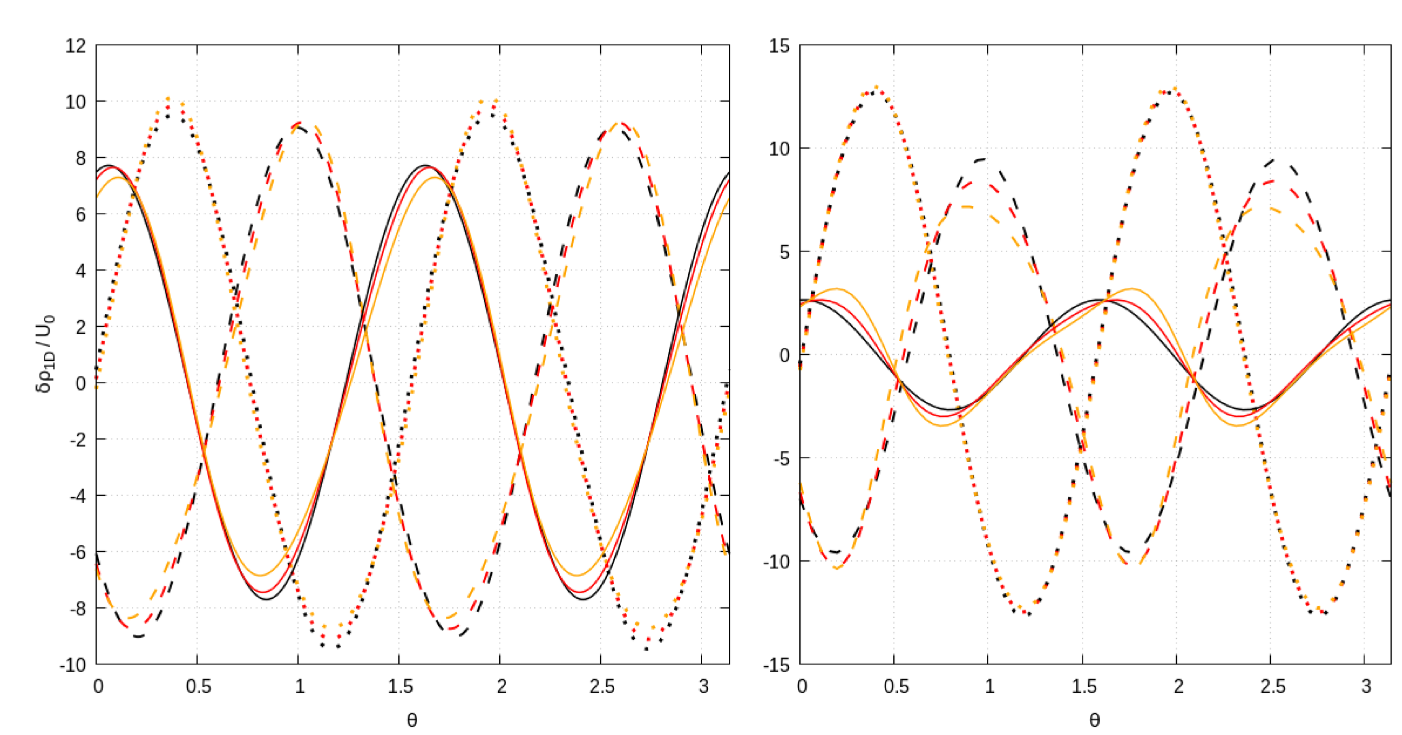

Fig. 3: Non-linear edge response of the FQH droplet of $N = 7$ bosons (left panel) compared with that of a IQH of $N = 14$ fermions (right).

The leading non-linear dynamics is captured by

$$\partial_t \rho = \left(v_\theta + \frac{2\pi c}{\nu}\rho\right)\partial_\theta \rho - \frac{\nu}{2\pi\hbar}\partial_\theta U, \tag{3}$$

which quantitatively describes the early-times non linear dynamics and qualitatively the shock-waves formation at finite times. It however misses the dispersive corrections which regularize the shock into ripples.

From this equation one sees that compression regions move faster than rarified ones, in qualitative agreement with the observed behaviour. This will eventually lead to the formation of a shock.

Finally, notice also that the "decay" of the density modulation is partially suppressed in the FQH state when compared to the IQH.

Outlook and conclusions

Even though our early results show no emergence of solitons carrying fractional charge, our investigations have shown that some wavebreaking dynamics is ultimately going on. We aim to understand whether our numerical results do support the Benjamin-Ono dynamics emerging from the chiral sector of the Calogero-Sutherland model in [3] for example by inspecting the quench-dynamics of localised perturbations and/or by analysing larger systems with the Monte Carlo.

Acknowledgements

We acknowledge financial support from the European Union via the FET-Open grant "MIR-BOSE" (n. 737017) and the H2020-FETFLAG-2018-2020 project "PhoQuS" (n.820392), from the Provincia Autonoma di Trento, partly through the Q@TN initiative, and from Google via the quantum NISQ award.

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