ANDREEV-BASHKIN DRAG ON 1D BOSE-HUBBARD RING

Daniele Contessi[1] , Donato Romito[2,3] , Matteo Rizzi[4,5] , and Alessio Recati[1,2]

[1]Dipartimento di Fisica, Università di Trento, 38123 Povo, Italy
[2]INO-CNR BEC Center, 38123 Povo, Italy
[3]Mathematical Sciences, University of Southampton,Highfield, Southampton, SO17 1BJ, United Kingdom
[4]Forschungszentrum Jülich, Peter Grünberg Institut (PGI-8), 52425 Jülich, Germany

"Collisionless drag for a one-dimensional two-component Bose-Hubbard model" arXiv:2009.09504

MOTIVATION

The **Andreev-Bashkin (AB) drag** is an effect discovered in the '70s that has no direct experimental measure so far. It is thought to play an important role in several physical multi-component systems such as **superconducting layers**, **neutron stars** and **ultracold atoms**.

Its dependence on the microscopic parameters has been obtained only in few cases. For this reason, it is very important to develop a model capable of indentifying the **most promising regimes in which the drag is enhanced** in order to guide an experimental measure.

HYDRODYNAMICS

Andreev and Bashkin corrected the work by Khalatnikov on the three-fluid hydrodynamics for the study of two-component mixture of ^{3}He and ^{4}He superfluids. They intruduced an **entrainment between two superfluids** (a drag) due to the interactions between different species (A and B) even without collisions.

They expanded the super-currents j_α as function of the superfluid velocities v_β (the greek letters run over A and B):

$$j_\alpha = \sum_\beta n^{(s)}_{\alpha\beta} v_\beta$$

introducing the matrix of the **superfluid densities** $n^{(s)}_{\alpha\beta}$

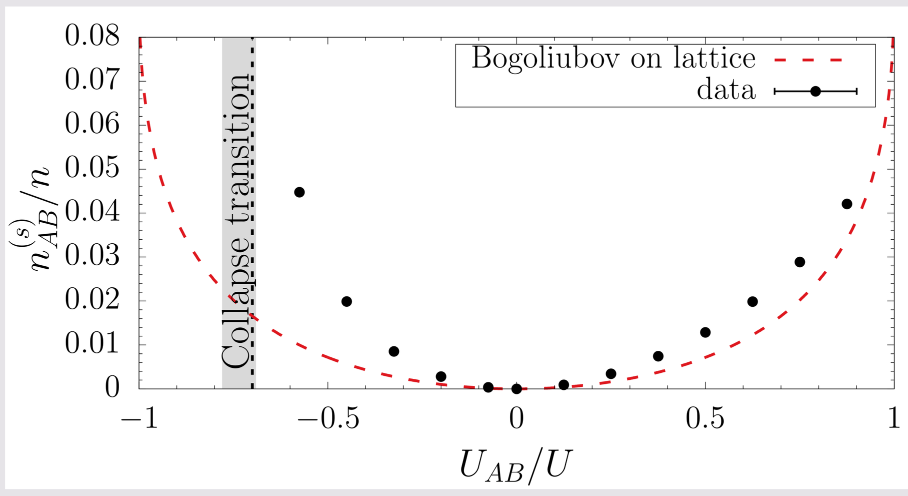
With respect to Khalatnikov, they add the off-diagonal terms of the matrix which correspond to the **superfluid amount of one species that accompains the motion of the other one due to the interaction between them**.

DRAG VS INTERACTION

From the currents, it is possible to obtain every element of the square matrix of the superfluid densities. The drag density is the off-diagonal part $n^{(s)}_{AB} = n^{(s)}_{BA}$

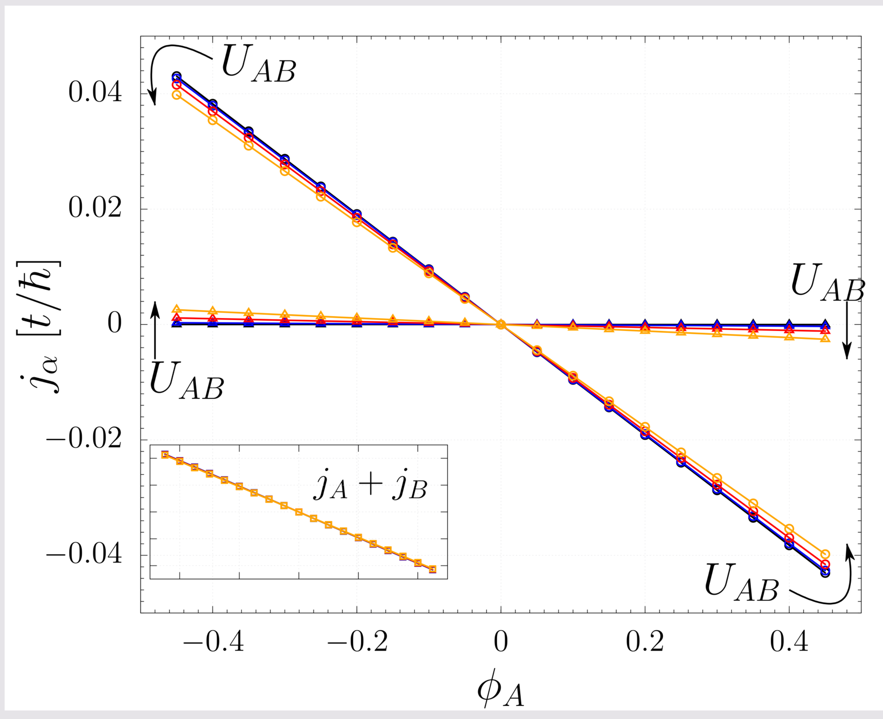
$$n^{(s)}_{\alpha\beta} = \lim_{\phi_\alpha,\phi_\beta \to 0} \frac{L\hbar}{4\pi} \frac{\partial j_\alpha}{\partial \phi_\beta}$$

In the figure, the **thermodynamical estimation for the normalized drag density** on the total density is represented as function of the interaction. The results are compared with the Bogoliubov prediction. There is an evident **asymmetry between the attractive and repulsive regime**. The attractive mixture experiences a much steeper growth of the drag as a function of the interaction strength. This substantial increase can be ascribed to pairing correlations.



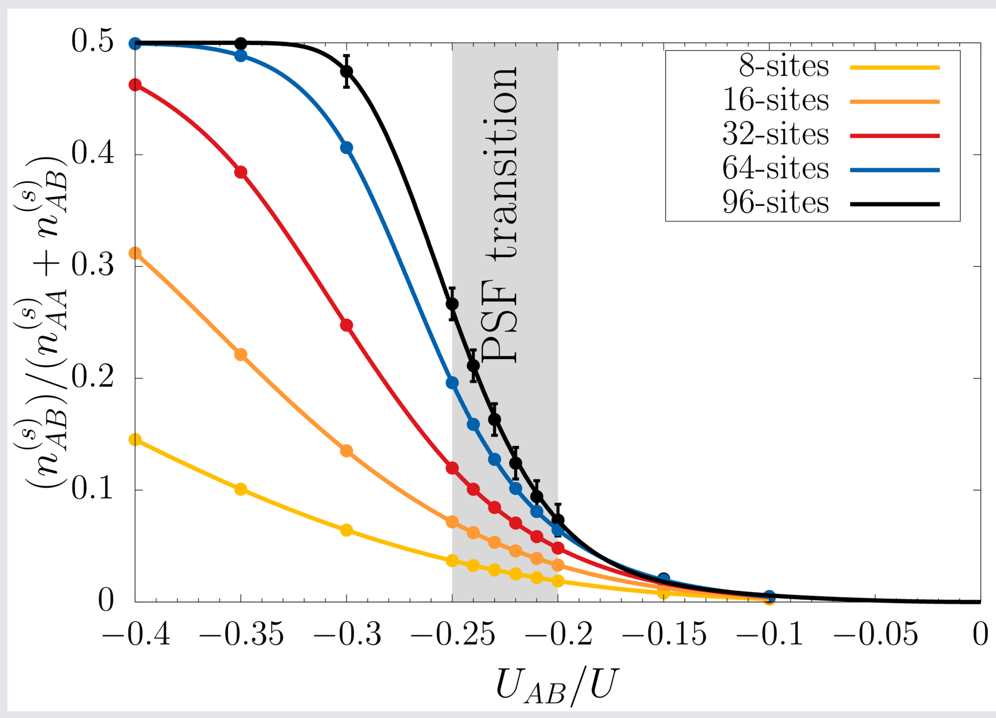
PERSISTENT CURRENTS

In the figure, the two currents are represented as a function of the flux for the A-species. The second flux is zero. The circles are the values of the A-current and the triangles are the ones of the B-current. As the interspecies interaction increases more and more B particles are involved in the super-flow, keeping the sum of the two currents constant at fixed flux due to the conservation of total momentum (in the inset). A non-zero B current for $\phi_B = 0$ means **the model is capable to describe the Andreev-Bashkin drag**.



PAIR SUPERFLUIDITY PHASE (PSF)

For some configurations of the system, the increase of the entrainment in the attractive regime is amplified by approaching the Pair Superfluid Phase. The two superfluids become a unique superfluid of dimers, where the flow of one component is accompanied by the same flow of the other, i.e. $n^{(s)}_{\alpha\alpha} = n^{(s)}_{\alpha\beta}$. The drag density, hence, saturates to a quarter of the total superfluid density as represented in the figure below for various sizes of the system. The drag is normalized to half of the total superfluid density as a function of the interaction. The shaded region indicates the occurence of the phase transition in the thermodynamic limit. The transition belongs to the Berezinskii -Kosterlitz-Thouless universality class.



MICROSCOPICAL MODEL

We consider a **two-species Bose-Hubbard Hamiltonian on a ring** of L - sites at **zero temperature**:

$$H = \sum_{x=1}^{L} \left\{ \sum_{\alpha=A,B} \left[-\left(\tilde{t}_\alpha b^\dagger_{x+1,\alpha} b_{x,\alpha} + \text{h.c.} \right) + \frac{U}{2} n_{x,\alpha}(n_{x,\alpha} - 1) \right] + U_{AB} n_{x,A} n_{x,B} \right\}$$

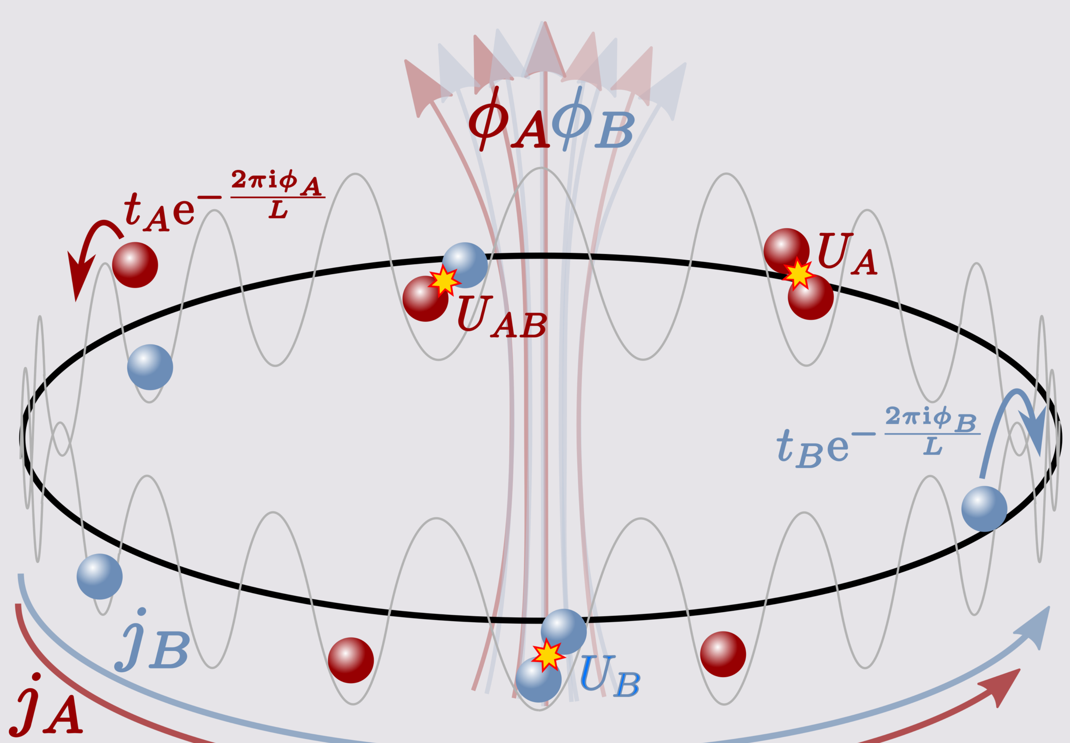
- $b^\dagger_{x,\alpha} (b_{x,\alpha})$ are the bosonic creation (annihilation) operators
- $n_{x,\alpha} = b^\dagger_{x,\alpha} b_{x,\alpha}$ is the number operator
- $\tilde{t}_\alpha = t e^{-i 2\pi\phi_\alpha / L}$ accounts for the hopping of the bosons between neighbouring sites
- U controls the **on-site interaction between particles of the same species** (always **repulsive** hence **positive**)
- U_{AB} controls the **on-site interaction between particles of different species** (**attractive or repulsive** according to sign)

The presence of the fluxes ϕ_α in the hopping terms, which can be interpreted as magnetic fluxes piercing the ring and pulling the bosons of the correspondent species, introduces **persistent currents** on the lattice. Their values are given by:

$$j_\alpha = \frac{2}{\hbar} \operatorname{Im} \langle \tilde{t}_\alpha b^\dagger_{x+1,\alpha} b_{x,\alpha} \rangle$$

A consequence of the drag effect is that if one switches only one of the two fluxes on, we observe the super-current of the correspondent species but also a current for the other one. In the latter, the bosons of the second species which are dragged by the first ones partecipate.

We numerically simulate the model with a **Tensor Network approach** by means of a MPS ansatz.



LUTTINGER LIQUID AND DRAG

In the two-superfluids phase the system is described by two coupled Luttinger Liquids hamiltonians in the density (in-phase) and polarization/spin (out-of-phase) channels. Combining such a description and a hydrodynamic approach based on density functional, the drag density appears in the constitutive relations of the Luttinger parameter of the spin channel:

$$K_S = \sqrt{t\pi^2 \left(n^{(s)}_{AA} - n^{(s)}_{AB} \right) \chi}$$

where χ is the spin susceptibility.

CONCLUSIONS

We provide a numerical estimation of the AB drag via a Tensor Network approach for a 1D Bose mixture on a ring lattice. The drag is enhanced for attractive interactions, in particular when close to the pair superfluidity transition. Our results are relevant for ultra-cold gases experiments where tens of atoms (e.g hyperfine states mixtures of *K* atoms, or *K-Rb* mixtures) are considered.