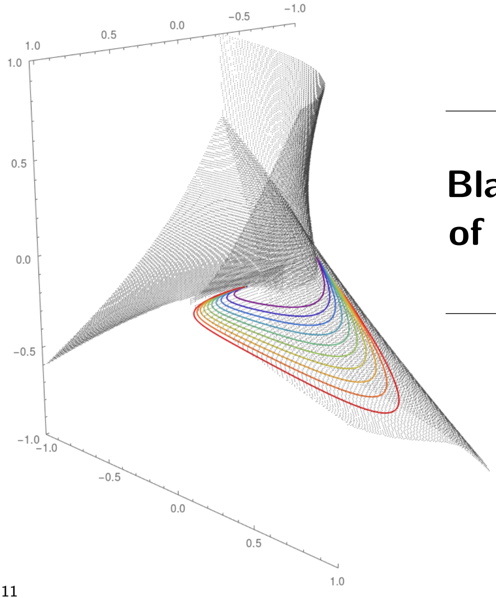

Trento, 2nd December 2020

Black holes, stars and exotic solutions of quadratic gravity

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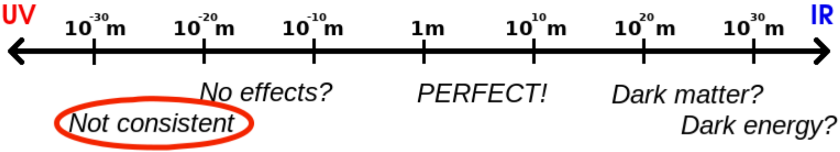
PhD Workshop 2020

Gravity and General Relativity: a scale problem



Coupling constant $[G] = E^{-2} \implies$ one-loop corrections $G^2 E^2_{\mathrm{cutoff}} \to \infty$

$$\mathcal{S}_{eff} = \int d^4x \sqrt{-g} \left[\underbrace{u_0 + u_1 R}_{GR} + u_2 R^2 + u_3 R^{\mu\nu} R_{\mu\nu} + u_4 R^3 + \dots \right] \tag{1}$$

Quadratic gravity: a classical model for quantum corrections

$$\mathcal{S}_{QG} = \int d^4x\sqrt{-g}\left[\gamma\, R + \beta\, R^2 - \alpha\, C^{\mu\nu\rho\sigma}\, C_{\mu\nu\rho\sigma}\right] \begin{cases} S = 2,\ m = 0 \\ S = 0,\ m_0^2 = \gamma/6\beta \\ \underline{S = 2,\ m_2^2 = \gamma/2\alpha} \end{cases} \tag{2}$$

PRO: renormalizable, general, IR limit of different theories

CON: negative energy states $\Longrightarrow$ non-unitary theory

Effective theory: classical solutions as first quantum corrections

Staticity, spherical symmetry:

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \tag{3}$$

$\sim$ **gravitational potential/time dilation factor** $\sim$ **length contraction factor**

Asymptotic flatness:

$$\begin{aligned} h(r) &\sim 1 - \frac{2\,M}{r} + 2\,S_2^{-}\frac{\mathrm{e}^{-m_2\,r}}{r} + S_0^{-}\frac{\mathrm{e}^{-m_0\,r}}{r} \\ f(r) &\sim 1 - \frac{2\,M}{r} + S_2^{-}\frac{\mathrm{e}^{-m_2\,r}}{r}(1 + m_2\,r) - S_0^{-}\frac{\mathrm{e}^{-m_0\,r}}{r}(1 + m_0\,r) \end{aligned} \tag{4}$$

Numerical methods

Two approaches:

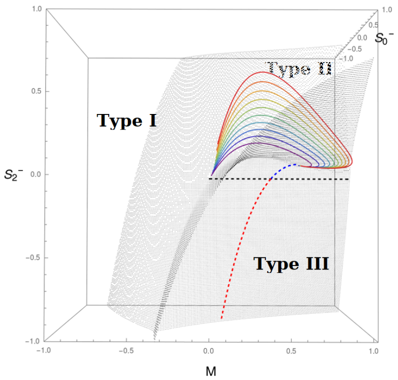
Integrate and classify the solutions *a posteriori* $\Longrightarrow$ IVP

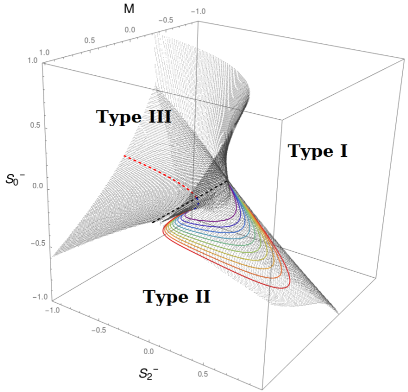
Choose a type of solution *a priori* and integrate $\Longrightarrow$ BVP

First method: easy and fast $\Longrightarrow$ scan of the parameter space

Second method (*shooting method*): more complex, more information on solutions $\Longrightarrow$ black holes, stars and wormholes

Parameter space of the theory





Dashed = Black holes, black is for Schwarzschild, red and blue for non-Schwarzschild

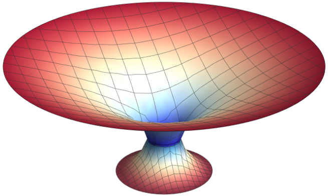
Rainbow = Non-vacuum solutions, different e.o.s. (polytropes)

Exotic solutions: type I, II, & III

Type I: divergent metric, naked curvature singularity, infinite mass in the origin

Type II: vanishing metric, naked curvature singularity, zero mass in the origin

Type III: wormholes $\implies$ $h(r) \to h_0, \; f(r) \to 0$ as $r \to r_W$

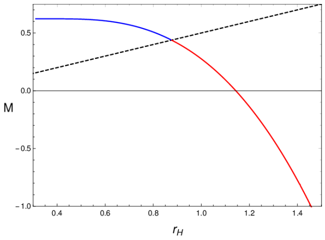


$$h_{cu}(r) \sim \mathrm{e}^{-m\,r}/r^2, \; 1/f_{cu}(r) \sim \mathrm{e}^{-m\,r}/r^2$$

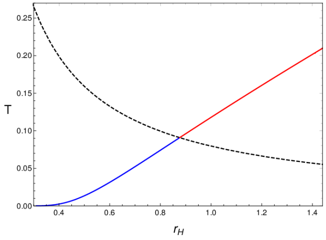
$\implies$ finite volume of the copy universe

Comparison with General Relativity: black holes

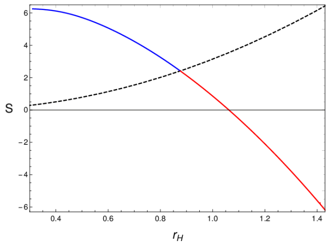
Thermodynamical properties of non-Schwarzschild black holes:



(a) ADM energy



(b) Hawking temperature



(c) Wald entropy

Negative entropy $\Longrightarrow$ signature of the ghost particle at classical level

Non vacuum solutions: a cure for the theory?

Vacuum solutions require $r \to 0$, *i.e.* $E \to \infty \implies$ breaking of effective theory

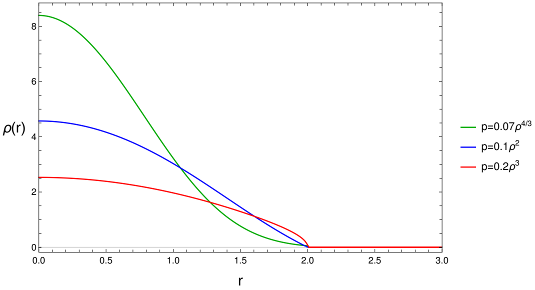
$$T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho(r) & 0 & 0 & 0 \\ 0 & p(r) & 0 & 0 \\ 0 & 0 & p(r) & 0 \\ 0 & 0 & 0 & p(r) \end{pmatrix} \qquad p(r) = k_0\, \rho(r)^{\Gamma} \tag{5}$$

$$M \sim \int_0^{R_*} \mathrm{d}s\, 4\,\pi\, s^2 \rho(s) \qquad S_2^- \sim \int_0^{\infty} \mathrm{d}s\, 4\,\pi\, s^2 \frac{\mathrm{e}^{m_2\, r} - \mathrm{e}^{-m_2\, r}}{m_2\, s} \left(\underline{2\,\rho(s) + 3\,p(s)}\right)$$

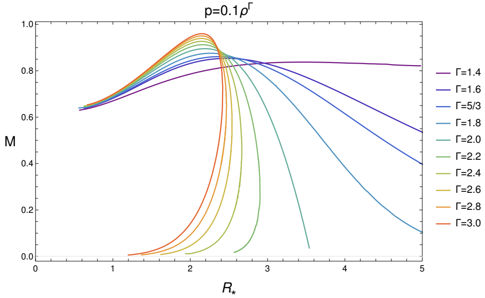
e.o.s. dependence

$$S_0^- \sim \int_0^{\infty} \mathrm{d}s\, 4\,\pi\, s^2 \frac{\mathrm{e}^{m_0\, r} - \mathrm{e}^{-m_0\, r}}{m_0\, s} \left(\underline{-\rho(s) + 3\,p(s)}\right) \tag{6}$$

No pathological behaviours!



(d) Density profile



(e) Mass-radius relation varying Γ

Constrained values $\alpha,\ \beta < 10^{60} \implies M_{phys} \lesssim \left(10^{-7} M\right) M_{\odot},\ \ R_{*,phys} \lesssim \left(10^{-7} R_*\right) km$

Summary and outlook

- General Relativity as effective theory $\Longrightarrow$ quadratic gravity as first correction
- Vacuum solutions have (at least) bizarre behaviours $\Longrightarrow$ signature of ghost particle
- Non vacuum solutions are sensible $\Longrightarrow$ possible dark matter candidate?

To do:

- Stability of the solutions
- Realistic dark matter model
  (Can they survive inflation?)

Thanks, and see you soon!