Anyonic molecules in atomic fractional quantum Hall liquids: a quantitative probe of fractional charge and anyonic statistics

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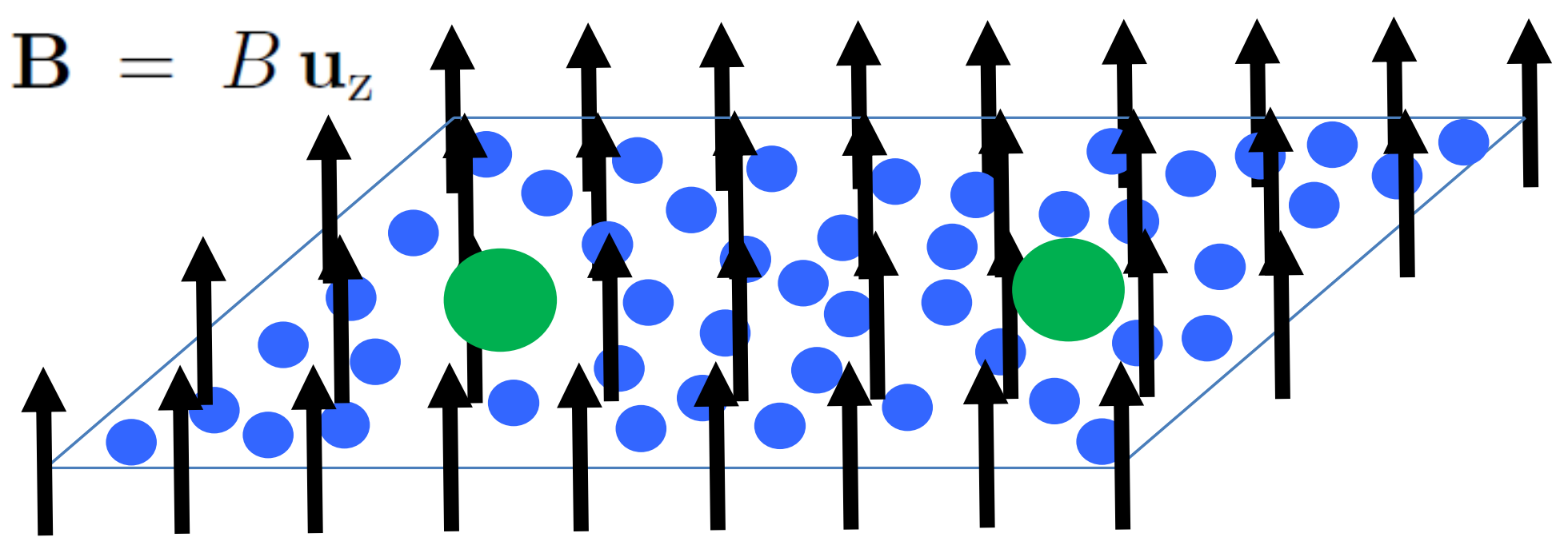
Introduction

We study the quantum dynamics of **massive impurities** embedded in a **fractional quantum Hall (FQH) liquid of cold atoms**. For suitable values of the atom-impurity interaction strength, each impurity can capture quasihole (QH) excitations of the FQH liquid, forming a bound molecular state. An effective Hamiltonian for such anyonic molecules is derived within the Born-Oppenheimer approximation, which provides renormalized values for their effective mass, charge and statistics. The renormalized mass and charge of a single molecule can be extracted from the cyclotron orbit that it describes as a free particle in a magnetic field. Signatures of the fractional statistics are anticipated in the interference pattern displayed by the angular cross section for a pair of indistinguishable colliding molecules.



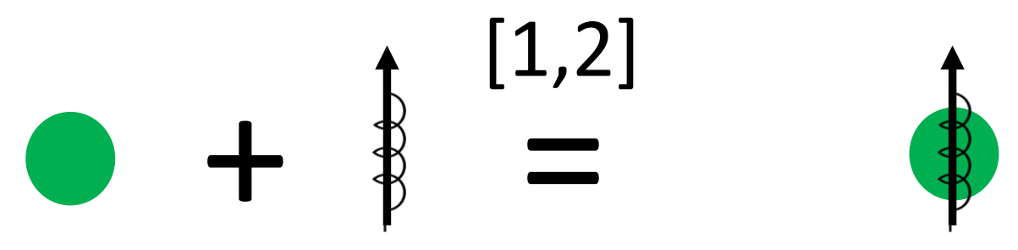
● *n* bath atoms (a) of mass *m* and charge *q*

● *N* impurities (i) of mass *M* and charge *Q*

$0 < \nu < 1$ is the Abelian filling fraction

Hamiltonian:

$$H = \underbrace{T_a + T_i}_{\text{Kinetic energies}} + \underbrace{V_{aa} + V_{ia} + V_{ii}}_{\text{Repulsive interactions}}$$



Impurity + QH = [1,2] **"Anyonic molecule"**

Born-Oppenheimer approximation

Total wavefunction: $\psi(\{\mathbf{r}_i\},\{\mathbf{R}_i\},t) = \boxed{\varphi^{(0)}_{\{\mathbf{R}_i\}}(\{\mathbf{r}_i\})}\boxed{\chi(\{\mathbf{R}_i\},t)}$

Atoms (fast) Impurities (slow)

Ground state of $H_{\mathrm{BO}} = T_a + V_{aa} + V_{ia}$ with energy $\epsilon^{(0)}_{\mathrm{BO}}$

Described by Laughlin wavefunction with QHs at impurities' positions

$$\varphi^{(0)}_{\mathbf{R}}(\mathbf{r}) = \frac{1}{\sqrt{\mathcal{N}}}\prod_{i=1}^{n}\prod_{j=1}^{N}(z_i - Z_j)\prod_{i<j}^{n}(z_i - z_j)^{1/\nu}e^{-\sum_{i=1}^{n}|z_i|^2/4\ell_B^2} \quad [3,4]$$

Dynamics governed by effective Hamiltonian $H_{\mathrm{eff}} = \left\langle \varphi^{(0)}_{\mathbf{R}} \middle| H \middle| \varphi^{(0)}_{\mathbf{R}} \right\rangle$

Renormalized mass

First correction beyond Born-Oppenheimer approximation [5]:

$$\varphi_{\mathbf{R}}(\mathbf{r},t) \simeq \varphi^{(0)}_{\mathbf{R}}(\mathbf{r}) + \varphi^{(1)}_{\mathbf{R}}(\mathbf{r},t)$$

Mass of anyonic molecule = impurity mass + correction (QH inertia):

$$\mathcal{M} = M + \Delta M$$

Mass correction given by:

$$\frac{1}{2}\Delta M_{\alpha\beta}v_\alpha v_\beta = \int d\mathbf{r}\,\varphi^{(1)*}_{\mathbf{R}}(\mathbf{r},t)\left[H_{\mathrm{BO}} - \epsilon^{(0)}_{\mathrm{BO}}(\mathbf{R})\right]\varphi^{(1)}_{\mathbf{R}}(\mathbf{r},t)$$

v is the impurity velocity

Correction to FQH ground state given by:

$$\left[H_{\mathrm{BO}} - \epsilon^{(0)}_{\mathrm{BO}}\right]\varphi^{(1)}_{\mathbf{R}}(\mathbf{r},t) = v_\alpha \nabla_\alpha \varphi^{(0)}_{\mathbf{R}}(\mathbf{r})$$

Figure of merit:

$$\frac{\Delta M}{M} \simeq \frac{m}{M}\frac{\omega_{\mathrm{cycl}}}{\Delta\omega_{-1}}$$

$\omega_{\mathrm{cycl}} = qB/m$ cyclotron frequency

$\Delta\omega_{-1}$ energy of 1st excited state (QH orbits impurity with angular momentum $\Delta L = -1$)

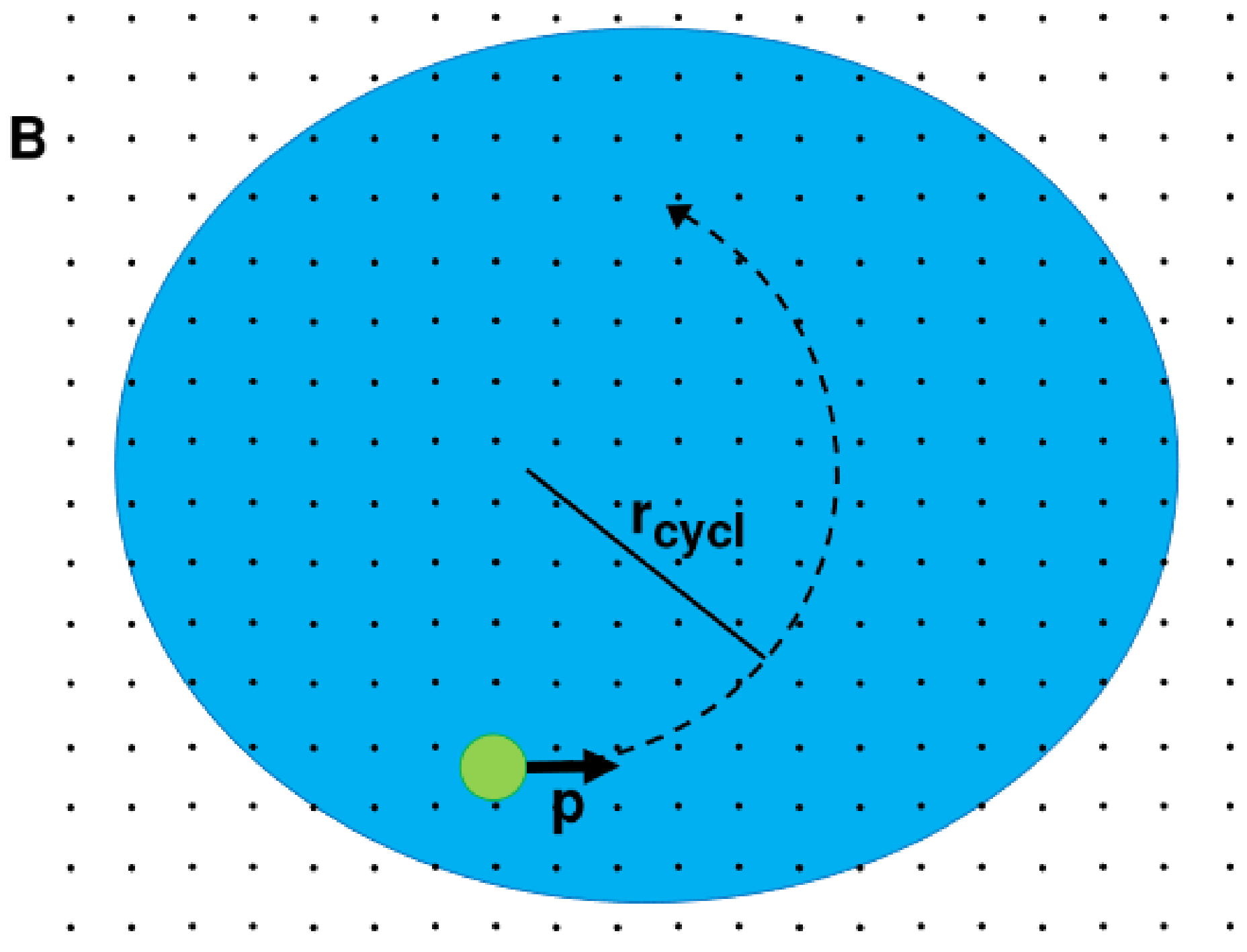
Renormalized charge

Effective Hamiltonian for single impurity:

$$H_{\mathrm{eff}} = \frac{\left[-i\nabla_{\mathbf{R}} - (Q - \nu q)\,\mathbf{A}(\mathbf{R})\right]^2}{2\mathcal{M}}$$

Fractional molecule charge = impurity charge + QH charge

$$\mathcal{Q} = Q - \nu q$$



Proposed experiment to measure fractional charge of anyonic molecule

(i) Apply deterministic kick $\mathbf{p} = \mathcal{M}\mathbf{v}$

(ii) Measure impurity's position at different times

$$r_{\mathrm{cycl}} = \frac{\mathcal{M}v}{\mathcal{Q}B}$$

(iii) Extract molecule charge

Renormalized statistics

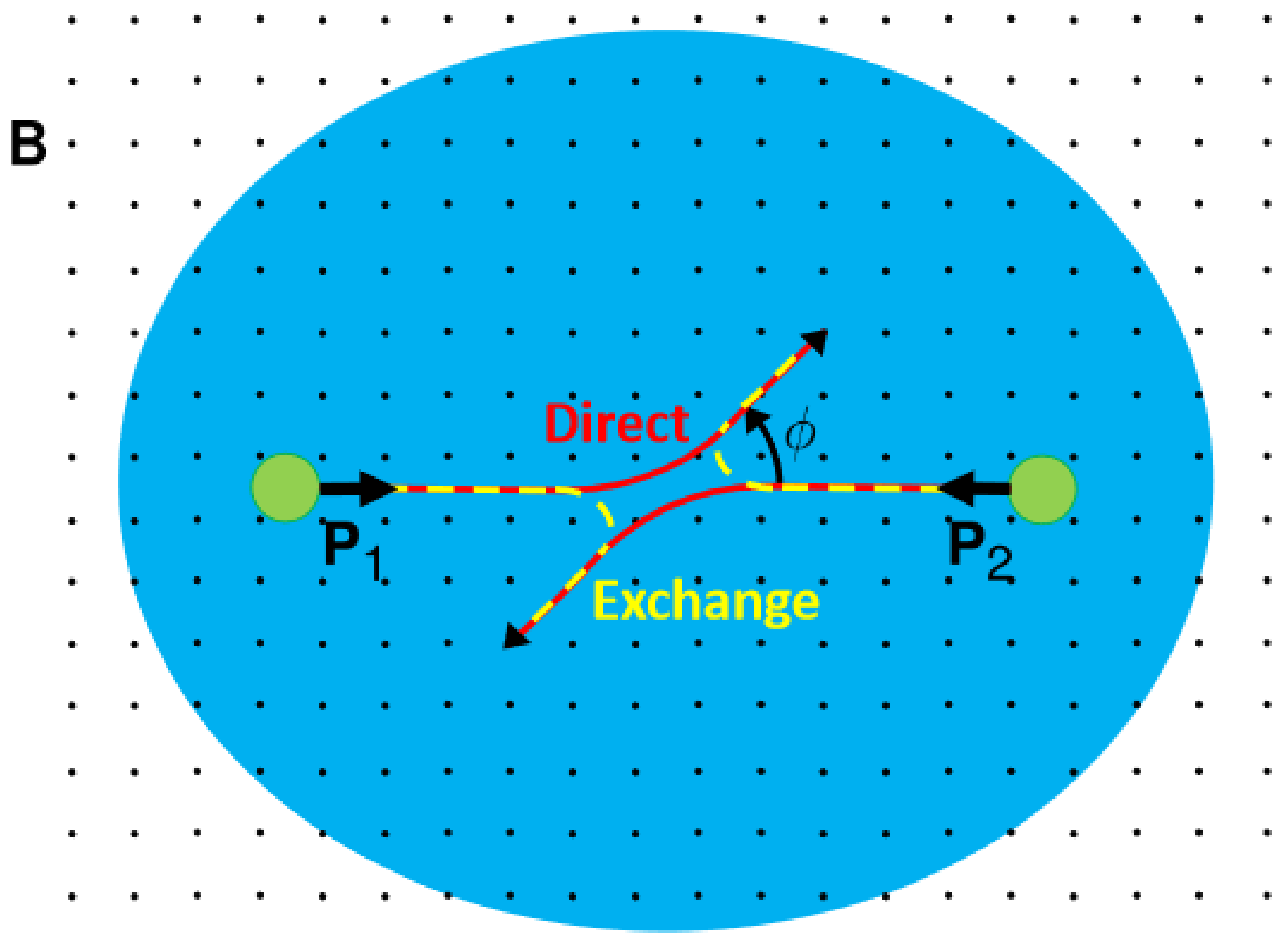
Effective Hamiltonian for two impurities:

$$H_{\mathrm{eff}} = \sum_{j=1}^{2}\frac{\left[-i\nabla_{\mathbf{R}_j} - \mathcal{Q}\mathbf{A}(\mathbf{R}_j) + \boxed{\mathcal{A}_{\mathrm{stat},j}(\mathbf{R})}\right]^2}{2\mathcal{M}} + V_{ii}(\mathbf{R})$$

Aharonov-Bohm interaction (contains info about molecules statistics)

Total statistical parameter α

- Bosonic impurities: α=ν
- Fermionic impurities: α=1+ν

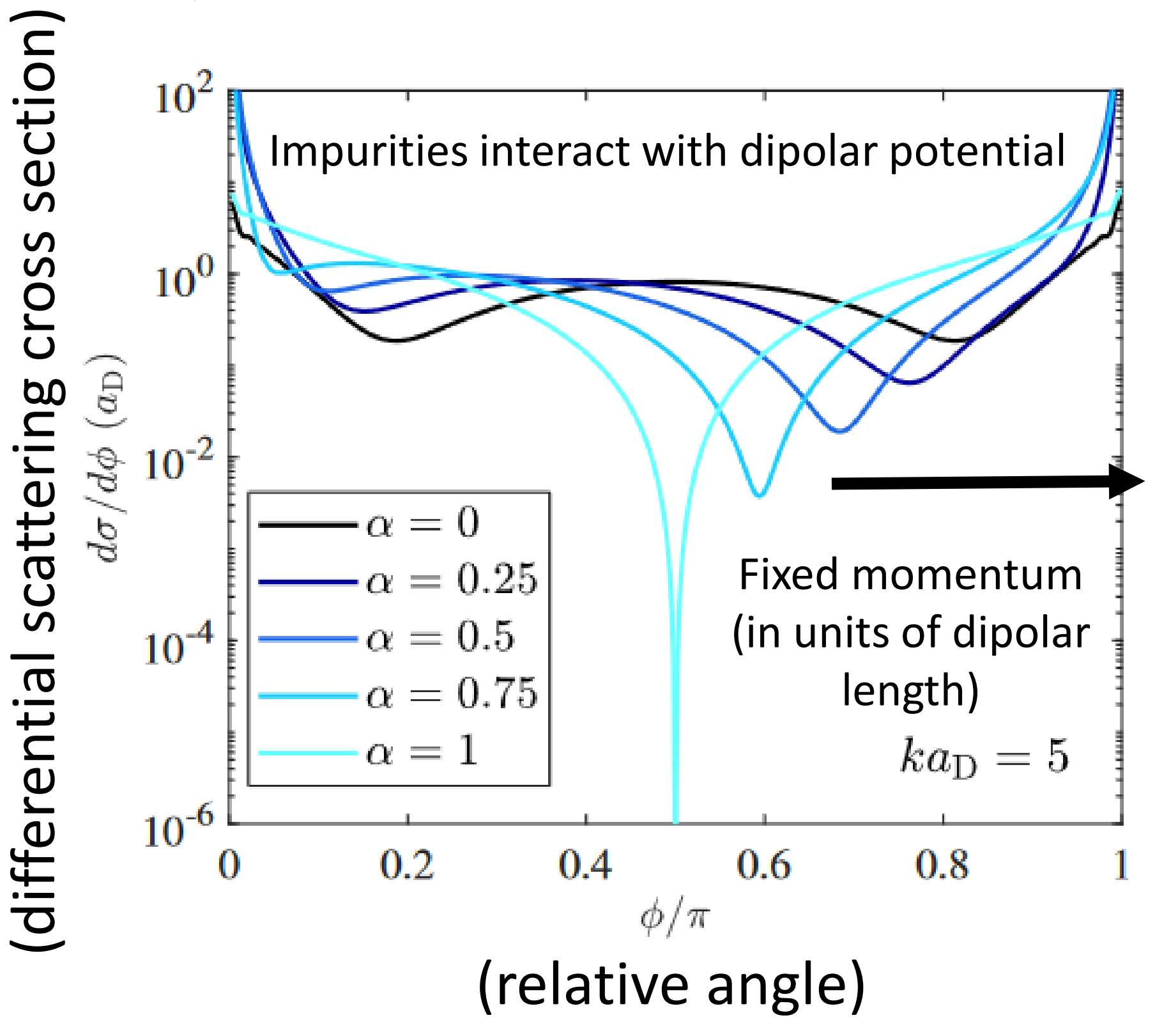


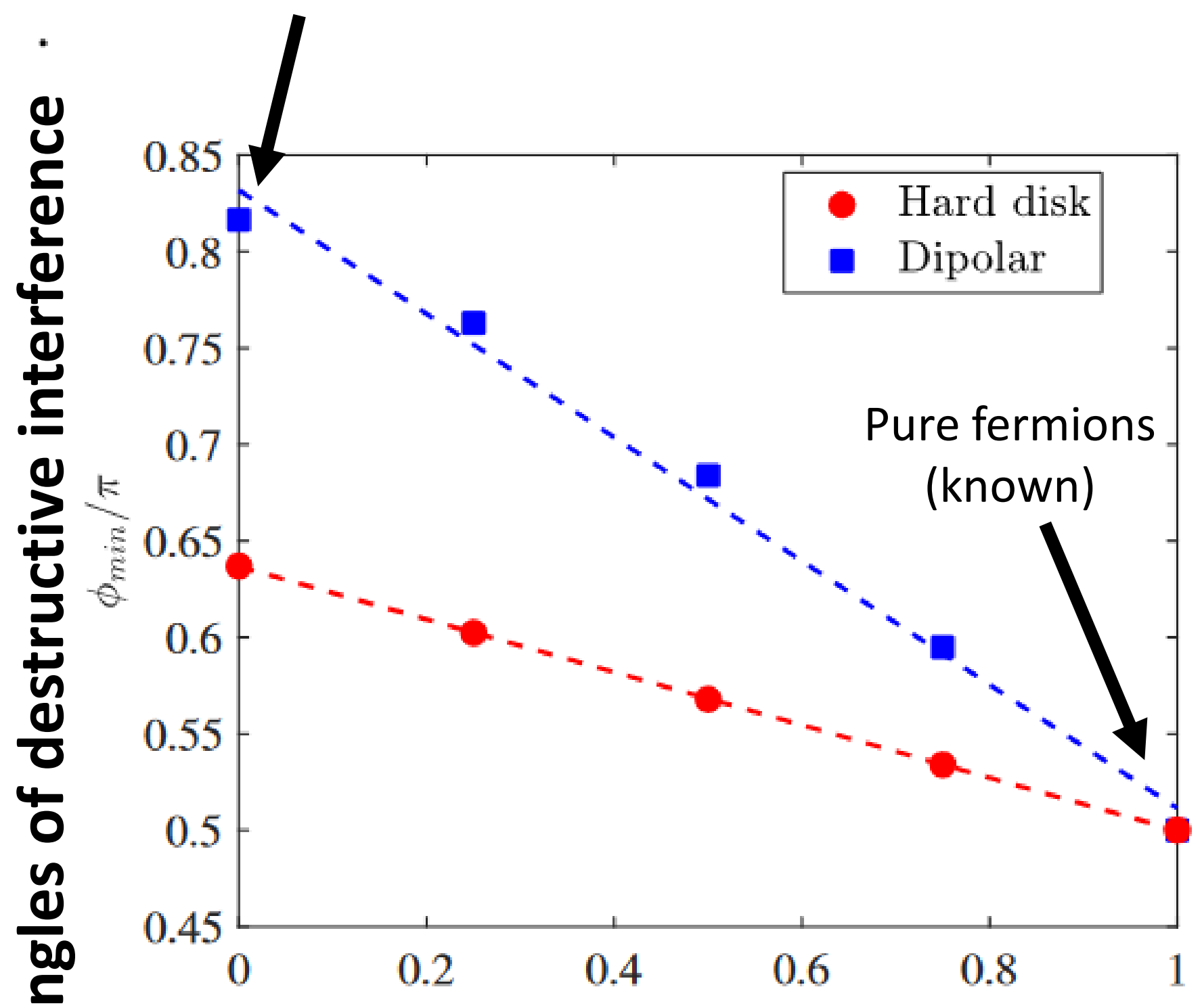
Proposed experiment to measure fractional statistics (bosonic impurities)

(i) Measure molecules' positions after scattering

(ii) Repeat and reconstruct differential scattering cross section

(iii) Repeat for impurities outside FQH (α=0)





(iv) Interpolate for α in FQH liquid

References

[1] Zhang et al, PRL 113, 160404 (2014)
[2] Lundholm & Rougerie, PRL 116, 170401 (2016)
[3] Laughlin, PRL 50, 1395 (1983)
[4] Tong, arXiv:1606.06687 (2016)
[5] Scherrer et al., PRX 7, 031035 (2017)

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