Fermi polaron and Rabi coupling

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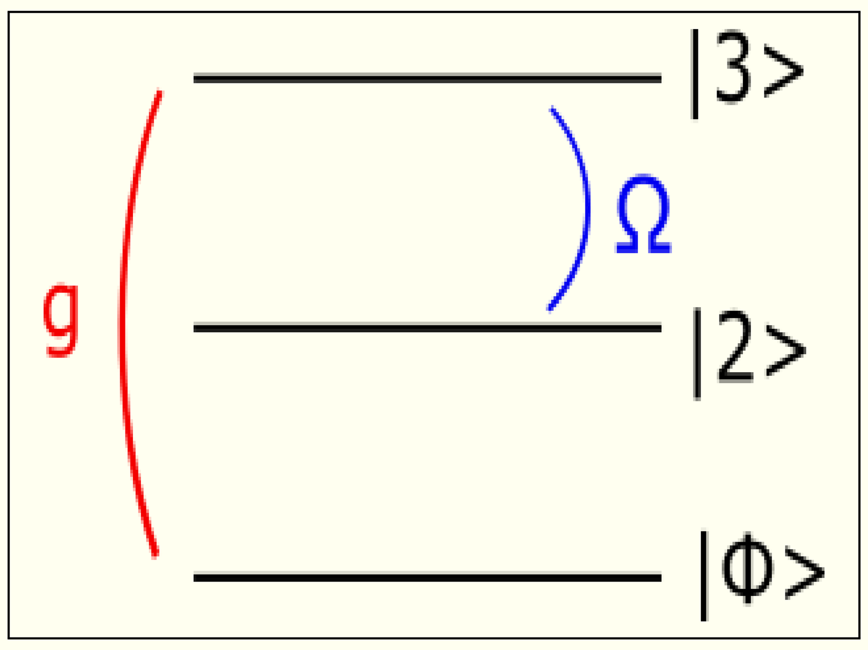
System and Hamiltonian

Study the dynamics of a system [1] composed of a bath $|\phi\rangle$ and 2 levels $|2\rangle$, $|3\rangle$ coupled by a Rabi frequency Ω. Level $|3\rangle$ is interacting with coupling constant g with the bath, while level $|2\rangle$ is non interacting.



The bath and levels are made of fermions of mass m. In level $|3\rangle$ there is polaron formation due to the interaction.
The total action of the system is:

$$S_{tot} = S_\phi + S_2 + S_3 + S_\Omega + S_{int}$$

$$S_\phi = \sum_{\mathbf{k}} \int_{-\infty}^{\infty} dt \bar{\phi}(\mathbf{k},t)(i\partial_t - \varepsilon_\phi(\mathbf{k}))\phi(\mathbf{k},t)$$

$$S_{2,3} = \sum_{\mathbf{k}} \int_{-\infty}^{\infty} dt \bar{\psi}_{2,3}(\mathbf{k},t)(i\partial_t - \varepsilon_{2,3}(\mathbf{k}))\psi_{2,3}(\mathbf{k},t)$$

$$S_\Omega = \Omega \sum_{vk} \int_{-\infty}^{\infty} dt \bar{\psi}_3(\mathbf{k},t)\psi_2(\mathbf{k},t) + h.c.$$

$$S_{int} = -g \sum_{vk} \int_{-\infty}^{\infty} dt \bar{\psi}_3(\mathbf{k},t)\bar{\phi}(\mathbf{k},t)\phi(\mathbf{k},t)\psi_3(\mathbf{k},t)$$

Is possible to write the action on the Keldysh contour to derive the equations that describe the dynamics of the populations in level $|2\rangle$ and $|3\rangle$.
The introduction of a new molecular field via an Hubbard-Stratonovich transformation makes possible to trace out the bath degrees of freedom.

Derivation of the equation of motion

We use some techniques illustrated in [2] to derive the equation of motions for the populations.

- Define the typical $2x2$ matrix on the Keldysh contour of the polaron self energy, Σ_{33}.
- Define the bare and dressed propagator matrices on the Keldysh contour.
- Use the Dyson equation to derive the retarded and advanced dressed propagators and population dynamics [3].

$$(\hat{G}_0^{-1} - \hat{\Sigma}) \circ \hat{G} = \mathbb{1}$$

- Definition of renormalized polaron dispersion relation dressed by the Rabi frequency

$$\omega_{p\,1,2}(\mathbf{k}) = \varepsilon(\mathbf{k}) + \frac{1}{2}\mathrm{Re}\Sigma^R_{33}(\mathbf{k}, \omega_{p\,1,2}(\mathbf{k})) \pm \frac{1}{2}\sqrt{(\mathrm{Re}\Sigma^R_{33}(\mathbf{k}, \omega_{p\,1,2}(\mathbf{k})) + 2\Delta)^2 + 4\Omega^2}$$

References

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Boltzmann equation for the populations

The dynamics of the populations can be described by 4 coupled **Boltzmann equations**: 2 equations for the populations of levels $|2\rangle$ and $|3\rangle$ and 2 equations for the coherence between the levels. They are indicated respectively as n_{22}, n_{33} and F_{23}, F_{32}.
In their complete form, the equations describes the complete dynamics of the system. Inside them all the polaron physics is present, e.g. attractive and repulsive branches, decaying between them and their reshaping due to the presence of the Rabi coupling.
In first approximation, we consider the timescale for the polaron formation much faster than the collisions due to the Rabi coupling. In this way is possible to reduce the Boltzmann equations to:

$$2\partial_t n_{22}(t,k) - i\Omega(F_{23}(t,k) - F_{32}(t,k)) = 0$$

$$2Z_p^{-1}(k)\partial_t n_{33}(t,k) + i\Omega(F_{23}(t,k) - F_{32}(t,k)) = 0$$

$$Z_t^{-1}(k)\partial_t F_{23}(t,k) - i\tilde{\Delta}(t,k)F_{23}(t,k) - 2i\Omega(n_{22}(t,k) - n_{33}(t,k)) = \mathrm{Im}\Sigma^R_{33}(k)F_{23}(t,k)$$

$$Z_t^{-1}(k)\partial_t F_{32}(t,k) + i\tilde{\Delta}(t,k)F_{32}(t,k) + 2i\Omega(n_{22}(t,k) - n_{33}(t,k)) = \mathrm{Im}\Sigma^R_{33}(k)F_{32}(t,k)\,,$$

with

$$Z_p^{-1}(k) = 1 - \partial_\omega \mathrm{Re}\Sigma^R_{33}(k) \quad Z_t^{-1}(k) = 1 - \frac{1}{2}\partial_\omega \mathrm{Re}\Sigma^R_{33}(k) = \frac{1}{2}(1 + Z_p^{-1}(t,k))$$

$$\tilde{\Delta}(k) = 2\Delta + \mathrm{Re}\Sigma^R_{33}(k)\,.$$

In the second approximation, we do not consider the conversion between the attractive and repulsive polaron branches. Indeed we want to verify that the main contribution to the dynamics is given by self energy on each branch and not by conversion phenomena.

Results and perspectives

It is possible to obtain a single equation for the dynamics of the difference of populations, $A(k,t) = n_{22}(k,t) - n_{33}(k,t)$. This equation is a third order differential equation:

$$\boxed{\partial_t^3 A(k,t) - D_2(k)\partial_t^2 A(k,t) + D_1(k)\partial_t A(k,t) - D_0(k)A(k,t) = 0}$$

$$D_2(k) = 2Z_t(k)\mathrm{Im}\Sigma^R_{33}(k)$$

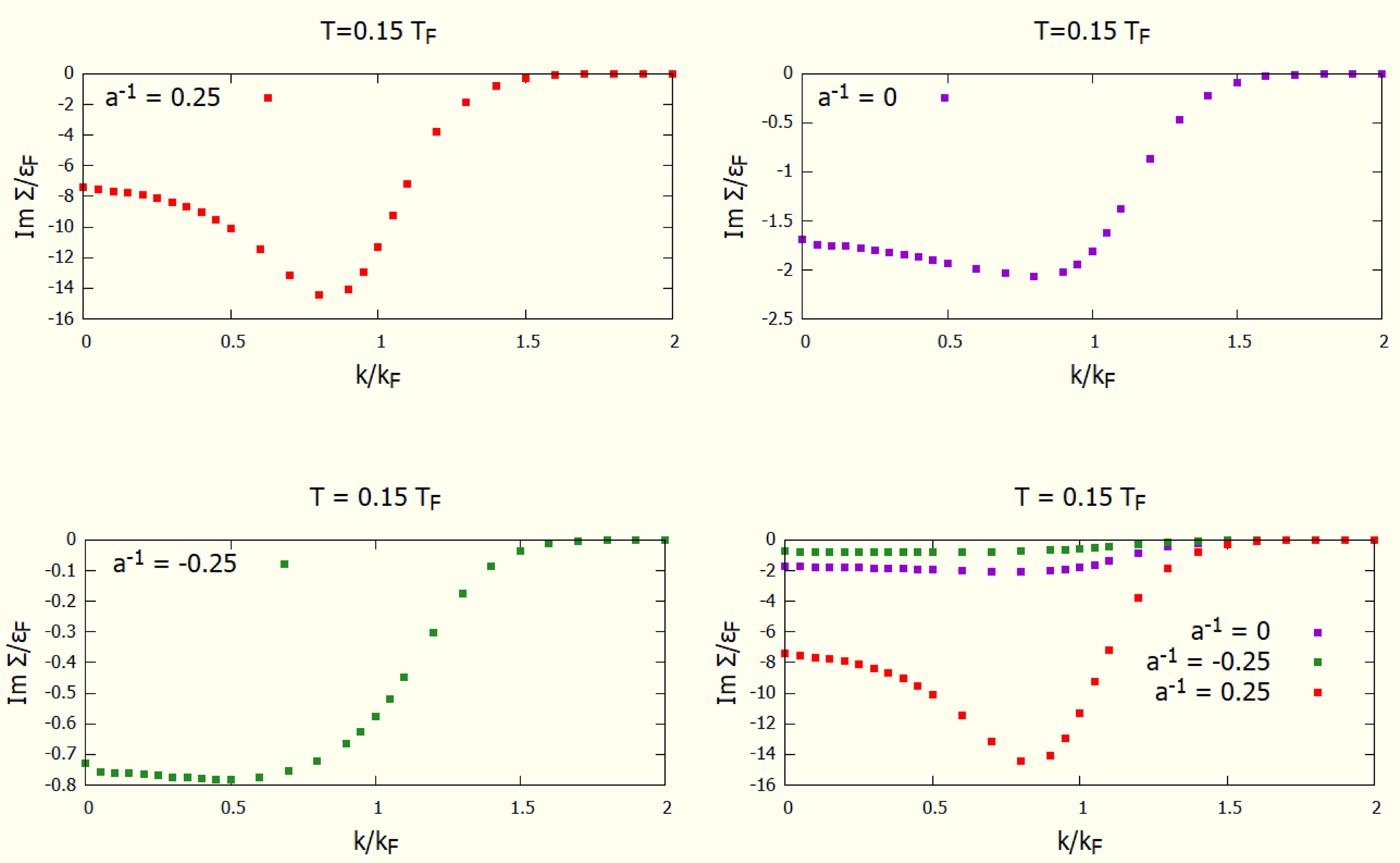
$$D_1(k) = Z_t^2(k)\left(\mathrm{Im}\Sigma^R_{33}(k)^2 + \tilde{\Delta}(k)^2\right) + 2\Omega^2 Z_t(k)(1 + Z_p(k))$$

$$D_0(k) = 2\Omega^2 Z_t(k)^2(1 + Z_p(k))\mathrm{Im}\Sigma^R_{33}(k)$$

The **imaginary** part of the retard self energy is crucial in describing the dynamics. Moreover, Keldysh treatment is necessary to compute it, while other methods underestimate it.
After all the approximation performed, the imaginary part is given by the expression:

$$\mathrm{Im}\Sigma^R_{33}(k) = -\frac{Z_p}{2}\int \frac{d^3q\, d^3q'}{(2\pi)^6}|G^R_\Delta(\omega_p(\mathbf{k}) + \varepsilon_\phi(\mathbf{q}-\mathbf{k}), \mathbf{q})|^2 \times$$
$$\times \delta(\omega_p(\mathbf{k}) + \varepsilon_\phi(\mathbf{q}-\mathbf{k}) - \varepsilon_\phi(\mathbf{q}') - \omega_p(\mathbf{q}-\mathbf{q}'))n_\phi^{\mathrm{eq}}(\mathbf{q}')n_{33}^{\mathrm{eq}}(\omega_p(\mathbf{q}-\mathbf{q}'), \mathbf{q}-\mathbf{q}')$$

The term G^R_Δ is the retarded molecular propagator and also this can be calculated in the Keldysh contour using an appropriate Dyson equation for the molecular propagator matrix.
We computed $\mathrm{Im}\Sigma^R_{33}(k)$ for different scattering lengths $a \propto g$ on the attractive branch of the polaron spectrum. Here there are the results for each scattering length and the comparison between them.



Future perspectives:

- Final implementation of the dynamics of population imbalance and comparison with experimental data [1]
- Extension to the repulsive branch.