Black hole with Immirzi hair

Simon Boudet

Physics PhD Workshop

University of Trento

December 2, 2020

Holst action and the Immirzi parameter ambiguity

The action of LQG is a first order formulation of the General Relativity action with the addition of the Holst term:

$$I[g_{\mu\nu}, \Gamma^{\rho}_{\mu\nu}] = \int d^4x\sqrt{-g}\left(\mathcal{R} - \frac{\beta}{2}\varepsilon^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}\right), \tag{1}$$

The Immirzi parameter β labels different, non unitary equivalent, quantum sectors of the theory constituting a quantum ambiguity. This issue has been addressed within different approaches:

- Studying the role of β in modified gravity theories, such as $f(\mathcal{R})$ theories;
- Promoting the Immirzi parameter to a new fundamental field: $\beta \to \beta(x)$.

We considered a general model featuring both of the above possibilities:

$$I[g_{\mu\nu}, \Gamma^{\rho}_{\mu\nu}, \beta] = \int d^4x\sqrt{-g}f\left(\mathcal{R} - \frac{\beta(x)}{2}\varepsilon^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu\rho\sigma}\right). \tag{2}$$

The Immirzi field: state of the art

The Immirzi field has been extensively studied considering its effects in cosmology, gravitational waves physics and its coupling with spinors, revealing interesting phenomenology like additional GW polarizations, bouncing cosmological solutions together with results regarding more fundamental problems such as the chiral anomaly and the CP problem.

However, investigations in the vacuum spherically symmetric sector of models featuring an Immirzi field are scarce in literature. This is mainly because of **no-hair theorems** which prevent the existence of black hole solutions with a non trivial radial profile for the scalar fields:

$$\text{Asymptotic flatness \& stationarity} \quad \Rightarrow \quad \beta(r) = const$$

We study the model with the aim of looking for analytical hairy solutions and possible effects of the Immirzi field both at a classical and semiclassical level.

Analytical hairy black hole solution

By evading no-hair theorems violating the asymptotic flatness assumption we found an asymptotically Anti-de Sitter topological black hole:

$$ds^2 = \Omega(r)\left[h(r)dt^2 + h(r)^{-1}dr^2 + r^2 d\sigma^2\right], \tag{3}$$

where

$$h(r) \overset{r\to\infty}{\sim} -\frac{\Lambda}{3}r^2, \tag{4}$$

and the Immirzi field surrounds it with scalar hair:

$$\beta(r) = \frac{e^{\beta_0/\sqrt{3}}(r+2m)^2 - e^{-\beta_0/\sqrt{3}}r^2}{2r(r+2m)}, \tag{5}$$

where β_0 is a free parameter and m the black hole mass.

Black hole thermodynamics

The analysis of the black hole thermodynamics via Euclidean path integral methods yields the black hole entropy:

$$S = [1 + g(\beta_h)]\,\frac{A}{4}. \tag{6}$$

Therefore a signature for a Immirzi field appears in the expression for entropy which instead reduces to the standard area law $S = A/4$ for a constant Immirzi parameter: $\beta \equiv const \Rightarrow g \equiv 0$.

The AdS asymptotics allows to study the system in the so called extended thermodynamic phase space, where a pressure term is provided by the cosmological constant $P = -\Lambda/8\pi$ and the thermodynamic volume V is its conjugate variable.

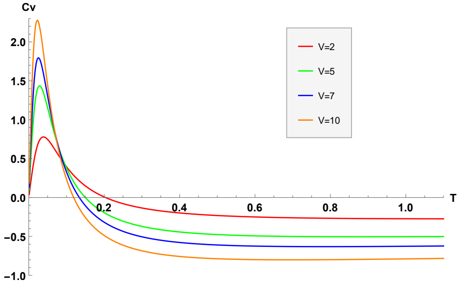
Most black holes satisfy the reverse isoperimetric inequality:

$$\xi = \left(\frac{3V}{\sigma}\right)^{1/3} \left(\frac{\sigma}{A}\right)^{1/2} \geqslant 1, \tag{7}$$

which is saturated by the Schwarzschild-AdS black hole. We observed a violation of the inequality: super-entropic black holes exist with $\xi \leqslant 1$. The recently proposed conjecture that:

all super-entropic black holes are thermodynamically unstable,

is confirmed also in this case since the specific heat at constant volume turns out to be negative.



Geodesics, autoparallels and projective invariance

Because of the first order formalism, $I[g_{\mu\nu}, \Gamma^{\rho}_{\mu\nu}]$, there is a non vanishing torsion tensor, $T^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} - \Gamma^{\rho}_{\nu\mu}$, which acquires dynamics sourced by the Immirzi field: $T^{\mu}_{\nu\rho} = T^{\mu}_{\nu\rho}(\partial_\mu \beta)$.
The two concepts of trajectories, namely

- **Geodesics** which minimize the proper distance between two spacetime points,
- **Autoparallels** being the "straightest" paths connecting two spacetimes points (The tangent vector parallely transported along the curve remain parallel to itself),

coincide in General Relativity but differ in presence of torsion. Whether of the two is the physical one is still an open debate in literature.
Both the action and field equations are invariant under projective transformations:

$$\Gamma^{\mu}_{\nu\rho} \to \Gamma^{\mu}_{\nu\rho} + \delta^{\mu}_{\nu} v_{\rho}. \tag{8}$$

<u>Only geodesics are invariant under projective transformations.</u>

Future perspectives

- Search for effects of the Immirzi field in cosmological observations via the DDR modification due to torsion;
- Pursue a more thorough analysis of the thermodynamic stability via non extensive methods;
- Carry on a quantum analysis of the model via implementation of suitably generalized Ashtekar-like variables.

Thank you for your attention!